Answers for class prep quiz on section 3.8, Stewart's Calculus (8th ed.)

1. **Answer:** (d). What is actually the case with $T(t)$ is that if T_s is room temperature and we let $y(t) = T(t) - T_s$, then $y(t) = Ce^{kt}$ for some constants C and k, or in other words, $T(t) = T_s + Ce^{kt}$.

2. **Answer:** (a). We know that $P(t) = P_0 e^{kt}$ (t in years), and we are given that $P(1) = 20000$ and $P(5) = 60000$. In other words,
$$20000 = P_0 e^k, \qquad\qquad 60000 = P_0 e^{5k}.$$
Dividing the second equation by the first, we get
$$3 = e^{5k}/e^k = e^{4k}.$$
Taking ln of both sides, we see that $\ln 3 = 4k$, or $k = \dfrac{\ln 3}{4}$. Therefore, since $20000 = P_0 e^{(\ln 3)/4}$, $P_0 \approx 15,200$, and the population at $t = 6$ is
$$P(6) = 15,000e^{6(\ln 3)/4} \approx 79000.$$

3. **Answer:** (a). Given $m(t) = m_0 e^{kt}$, to say that the half-life is h means that $m(h) = \frac{1}{2}m_0$, which means that
$$\frac{1}{2}m_0 = m_0 e^{kh},$$
and so $e^{kh} = \frac{1}{2}$, which means that $k = \dfrac{\ln(1/2)}{h}$. In fact, this formula is always true for radioactive decay, though the half-life h may vary.

4. **Answer:** (b). Let $y(t) = T(t) - T_s = T(t) - 70$. We know that $y(t) = y_0 e^{kt}$ for some constants y_0 and k. Since $T(0) = 98$, $y_0 = y(0) = 28$, and since $T(1) = 93$, $y(1) = 23$ and
$$23 = 28e^k,$$
which means that $k = \ln(23/28)$. Therefore,
$$T(3) = 70 + y(t) = 70 + 28e^{(\ln(23/28))3} \approx 85.5°F.$$